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12 Gamma estimation

Gamma in video systems is a numerical parameter related to the mapping from video signal values to light at the display screen. Video systems use various signal coding schemes, from analog voltage to 8-bit or 10-bit digital codes, with or without provision for footroom ("blacker-than-black") and headroom ("whiter-than-white"). We'll simplify by assigning reference black the value 0 and reference white the value 1; you can call these 0% and 100%.

7.5-unit black level ("setup") is a relic of 525-line SD analog video. It has no place in 625, no place in digital video, and no place in HD.

Luminance properly refers to a specific spectral weighting associated with the lightness sensation of vision; it can be measured by an instrument such as a photometer or a colorimeter. *R'G'B'* signals are presented to a colour display, and red, green, and blue light is produced. We are interested in the luminance of the red, green, and blue components. The colour science term for the linear-light *RGB* quantities, relative to their reference values, is *tristimulus values.*

Many video engineers and textbooks mistakenly use the word *luminance* to refer to the video signal *Y'*, which I argue should be called *luma.* Luminance can't be computed from luma alone.

A display's mapping of signal to light is described by an *electro-optical conversion function* (EOCF); it characterizes light produced by the display at the display surface. Displays are typically viewed in environments having a small amount of ambient illuminance. Upon diffuse reflectance at the display surface, ambient illuminance contributes to luminance. We exclude that light from the EOCF.

It's a sad story of studio practice that for the last 5 decades we haven't had a proper standard for gamma. However, a few months ago the ITU-R agreed that gamma of 2.4 approximates current studio mastering practice. In high-end video (such as home theatre), we seek to mimic the image appearance at mastering. The desired EOCF can be expressed as a pure power function of the form $L(V) = V^{2.4}$.

The ITU-R's 2.4 value is appropriate for a 100 nt display in a dark viewing condition. In Poynton's Vector issue 3, I discussed how that value should change for brighter displays or brighter viewing conditions.

Now, some math. A pure power function, when plotted in log-log coordinates, plots a straight line whose slope is the value of the exponent. In our case, log of tristimuli versus log of video signal plots a straight line whose slope ($^{rise}/_{run}$) is the value of gamma. However, many displays exhibit departures from pure power functions, and if the departures are large enough, visual disturbances might result. We'd like to characterize the overall gamma.

We can estimate gamma with just two measurements. As the "run" along the *x*-axis, use video signal values 0.1 and 1, a decade – that is, factor of ten – apart. As the "rise" on the *y*-axis, take the logs of the corresponding luminances L(0.1) and L(1). We would expect these two luminance values to be about 2.4 decades apart – that is, we expect L(0.1) to be about $10^{-2.4} \cdot L(1)$, that is, about 0.004 of reference white.

You can estimate gamma as the log of the ratio of those luminance values, or these additional two ratios that I will describe in a moment:

Eq 12.1

$$\gamma \approx \log_{10} \frac{L(1)}{L(0.1)} \approx \log_{10} \frac{L(0.8)}{L(0.08)} \approx \mathbf{\frac{5}{3}} \log_{10} \frac{L(0.8)}{L(0.2)}$$

Even in home theatre displays, reference white luminance is liable to be depressed a little owing to saturation. Effects of saturation on the gamma estimate can be reduced by using video signal values 0.08 and 0.8, still a decade apart: Gamma can be estimated as the log of the ratio L(0.8) to L(0.08), the second case in Equation 12.1. However, video code 0.08 is getting uncomfortably close to black level. The third case in Equation 12.1 takes a $^3/_5$-decade range of video signals ($0.6 = \log {}^{0.8}/_{0.2}$) and estimates gamma as five-thirds times the log of the corresponding luminance ratio.

Home theatre calibration software has historically calculated gamma not just from two measurements but ten, taking this average:

Eq 12.2

$$\gamma = \frac{1}{N-1} \sum_{i=1}^{N-1} \frac{\log_{10} \dfrac{L\left(\frac{i}{N}\right)}{L(1)}}{\log_{10} \frac{i}{N}}$$

The log of zero is minus infinity: We omit video signal value 0.

This equation is not as scary as it seems at first glance. Typically, we take ten display luminance measurements, at video signals 0.1, 0.2, …, 1.0. *N* will then be 10; the leading factor in Equation 12.2 is then $^1/_9$. The summation is over nine samples; the average gamma is ultimately just the average of quantities related to those nine samples.

Within the average are nine ratios; each numerator is a log, and each denominator is a log. The ratio is $^{\text{rise}}/_{\text{run}}$, that is, the slope. The fact that logarithms are taken in both the numerator and denominator reflects computation of the slope in log-log coordinates.

The numerator reflects luminance; we take the log of the ratio of each luminance step $L({}^{i}/_{N})$ with the luminance of reference white L(1). The denominator reflects the video signal; we take the log of each step $^{i}/_{N}$ of the video signal. The reference white video signal has the value 1, we need not divide $^{i}/_{N}$ by that. The quantities being "logged" are all less than one; the logs are negative, and each ratio is positive.

In brief, we are averaging the slopes of nine lines in log-log space. Denoting each measurement point [*x*, y], each line joins [$\log {}^{i}/_{N}$, $\log L({}^{i}/_{N})$] with the reference white point [log 1, log L(1)].

You might now suspect a problem: The reference white luminance L(1) occurs nine times in the sum. That single luminance value has nine times the influence on the average gamma than each of the other nine values. If you seek to obtain a meaningful display gamma estimate using this formulation, you must avoid saturation at 100% video.

Researchers working toward the adoption of the ITU-R studio EOCF standard didn't establish gamma using the approach of Equation 12.2. Instead, they used a numerical optimization technique to estimate the parameters of an equation modelling the EOCF. Such an optimization technique should, in my view, be adopted by home theatre practitioners. Such a proposal is likely to be the subject of a future issue.